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ON-BOARD BEARING PROGNOSTICS IN AIRCRAFT ENGINE : ENVELOPING ANALYSIS OR FFT? (PREPRINT)

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Metals, Ceramics and NDE Division

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ABSTRACT:

Prognostics of rolling element bearing mandates detecting bearing defect signatures as early as possible, so that corresponding maintenance can be scheduled and catastrophic machine breakdown can be avoided. Defects can occur in any of the bearing parts, inner and outer race, cage and rolling elements. It is possible to identify the defective component of the bearing based on the specific vibration frequencies that are excited. However, the pattern of vibration spectrum changes as bearing deteriorates through different stages. Depending on which failure stage the bearing is in, different techniques should be adopted to seek signature in different frequency ranges. Techniques such as enveloping analysis that performs in high frequency region apparently requires higher data sampling rate and more expensive data acquisition hardware than those analysis conducted in low frequency region.

This paper compares two popular rolling element bearing diagnostics techniques, spectrum analysis in bearing characteristic frequency range and enveloping analysis in high frequency range, using test data from an aircraft engine test rig. The objectives are to compare the techniques in terms of the time of detection and data requirement, and provide guidance for technology adoption in further field deployment. Results demonstrate that enveloping analysis is able to detect bearing defect much earlier than the spectrum analysis, but it requires a higher data sampling rate. Bearing defect characteristic frequency eventually shows up in low frequency spectrum at the late stage of the failure and it is modulated by other harmonics such as shaft unbalance. From the practical deployment point of view, a final decision of technology adoption should be made based on trade-off analysis of hardware requirements and tolerance of detection latency.

INTRODUCTION:

Bearing is a critical component on an aircraft engine. Detecting its defect at the earliest possible stage and assessing its damage condition in real time have profound meaning to operational safety and mission success. Bearing prognostics problem can be divided into three sub problems, a) defect detection - detect the existence of any defect as robust and early as possible, b) defect assessment - quantitatively assess current defect (damage) level, and c) life prediction - forecast defect propagation rate under any given operational scenarios[1]. The success of life prediction relies on accurate defect detection and assessment, and more importantly good understand of fault propagation theory. In this paper, discussions will be mainly focused on techniques for defect detection.

Vibration signature is a widely used indication of bearing defect. Features from various signal processing techniques [2] such as Fourier transform, Hilbert transform [3], and Wavelets[4][5] can be useful in detecting and categorizing incipient faults. Although its effectiveness is influenced by the loading condition, operational speed, and background noise level etc, a carefully processed vibration feature can not only provide a precursor for early warning, but also a unique capability for fault isolation if it factors into bearing geometric information and operating speed etc[6][7][8].

The pattern of vibration spectrum changes as bearing deteriorates through different stages [9]. Depending on which failure stage the bearing is in, different techniques should be applied to seek signature in different frequency ranges. Techniques such as enveloping analysis that performs in high frequency region apparently requires higher data sampling rate and more expensive data acquisition hardware than those analysis conducted in low frequency region. Hardware complexity and computational overhead might not a big issue for condition monitoring of industrial rotating machinery where a permanent system is installed or periodical inspections are conducted by handheld devices. However, in an aircraft engine on-board prognostic application, the situation is quite different. Not only the weight of the added on instruments need to be cautiously controlled, but also the hardware complexity and computational demand have to be

carefully considered. Computational power ought to be carefully prioritized and distributed among multiple processes to ensure the safety of the critical tasks such as flight and engine controls.

This paper aims to addresses the practical issue of which technique is more suitable for on-board bearing prognostics of aircraft engines. A trade-off between hardware complexity and detection latency is laid out based on reviewing of bearing failure theory and lab experiment, which provides a good reference for technology adoption of on-board prognostic system design.

This paper is organized as follows. In section II and III, theories of bearing defect characteristic frequency and four stage failure model are reviewed. In section IV, amplitude modulation theory is reviewed. Two different types of amplitude modulation phenomena in bearing vibration signal are presented in section V. In section VI, experimental results are presented to demonstrate the differences between the enveloping analysis and the FFT approaches. Finally conclusions are given in section VII.

BEARING DEFECT CHARACTERISTIC FREQUENCY

Rolling element bearings produce vibration excitation forces at specific frequencies dependent on the bearing geometry and rotation speed [10]. These vibration frequencies are called bearing tones or bearing characteristic frequencies. If the bearing dimensions are known, the individual bearing characteristic frequencies can be calculated precisely. Generally, there are four frequencies associated with a rolling element bearing:

- Cage frequency or Fundamental Train Frequency (FTF);

$$f_{FTF} = \frac{1}{120}\left[N_{OR}\left(1+\frac{d}{D}\cos\theta\right)+N_{IR}\left(1-\frac{d}{D}\cos\theta\right)\right] \quad (1)$$

- Rolling element frequency (BSF)

$$f_{RE} = \frac{D}{120d}\left(1-\frac{d}{D}cos\,\theta\right)\left(1+\frac{d}{D}cos\,\theta\right)\left|N_{OR}-N_{IR}\right| \quad (2)$$

- Outer raceway frequency (BPFO).

$$f_{OR} = \frac{n}{120}\left(1-\frac{d}{D}\cos\theta\right)\left|N_{OR}-N_{IR}\right| \quad (3)$$

- Inner raceway frequency (BPFI)

$$f_{IR} = \frac{n}{120}\left(1+\frac{d}{D}\cos\theta\right)\left|N_{OR}-N_{IR}\right| \quad (4)$$

Where d is the diameter of the ball/roller, D is the bearing pitch diameter, n is number of balls or rollers, θ is the contact angle, and N_{OR}, N_{IR} are rotating speed (RPM) of the outer race and inner race respectively.

FOUR STAGE FAILURE MODEL [9]

The four stage rolling element fault propagation model divides the bearing deterioration process into four consecutive stages. During the initial stage, it is just a high-frequency vibration, after which bearing resonance frequencies are observed. During the third stage, discrete defect frequencies (BPFO etc.) can be seen, and in the final stage high-frequency random noise is observed, which keeps broadening and rising in average amplitude with increased fault severity.

The FFT spectrum for bearing defects can be split into four zones (A,B,C and D)

- Zone A: machine rpm and harmonics zone
- Zone B: bearing defect frequencies zone
- Zone C: bearing component natural frequencies zone
- Zone D: high frequency defection zone

Stage 1

In the first stage, indication of bearing wear show up in the spike energy and ultrasonic frequency region (Zone D) that can only be evaluated by high-frequency detection techniques such as Spike energy and Shock Pulse Method etc [11]. The raceways or rolling elements of the bearings may not show any identifiable defects.

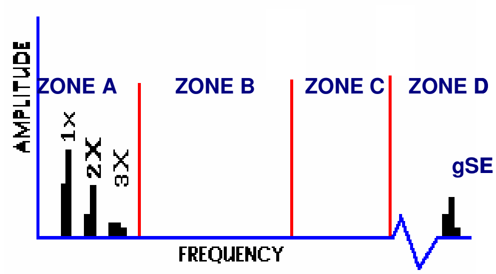


Figure 1. Stage 1 spectrum, defect signature on Zone D

Stage 2

In stage 2, the fatigued raceways begin to develop minute pits and generate signal associated with natural resonance frequencies of the bearing parts as bearing defects begin to “ring” the bearing components. A notable increase in Zone C and D region signals is associated with this stage. Depending on the severity of the defect, sideband frequencies (bearing defect characteristic frequencies) might appear above and below the natural frequency peak. Enveloping analysis[3], which includes a series of processes (band-pass filtering, rectification, Hilbert transformation, and spectral analysis), is usually used in this stage to demodulate the signal and extract the bearing defect characteristic frequency components.

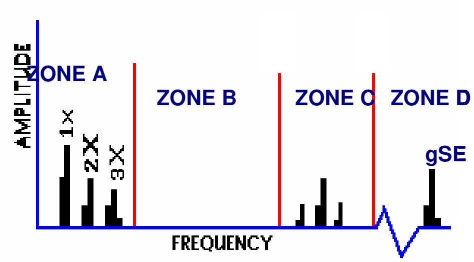


Figure 2. Stage 2 spectrum, defect signature on Zone C and D

Stage 3

As the bearing wear progress to the third stage, the discrete bearing defect characteristic frequencies start to become visible in the FFT spectrum (Zone B). Harmonics of these frequencies may be present depending upon the quantity

of defects and their dispersal around the bearing races. The harmonic frequencies will be modulated, or side banded, by the shaft speed. It is usually advised to replace the bearing at this stage since the remaining useful life after this stage could be less than 1% of its average life.

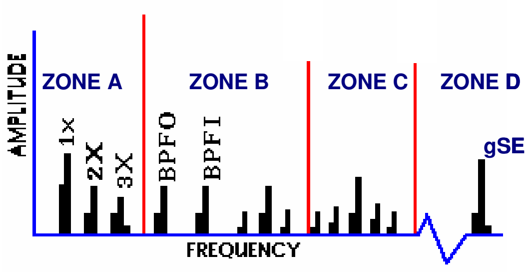


Figure 3. Stage 3 spectrum, defect signature on Zone B

Stage 4

In the final stage, the pits merge with each other, creating rough tracks and spall of the bearing raceways or/and rolling elements. The bearing fundamental frequencies will decline and be replaced with random noise floor or "hay stack" at higher frequencies. Zone D signal levels will actually decrease with a significant increase just prior to failure. The bearing will have excessive vibration. It will be hot and making lots of noise. If it is allowed operating further, a cage failure will be happening soon.

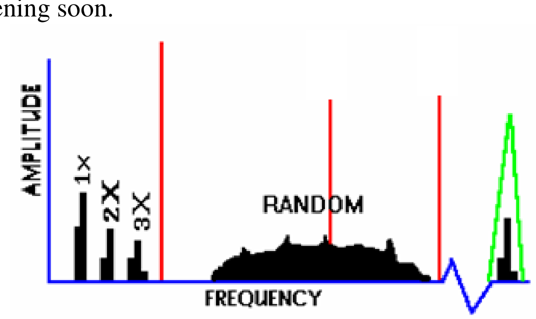


Figure 4. Stage 4 spectrum, noise floor rises

AMPLITUDE MODULATION

Amplitude modulation (AM) is a technique used in electronic communication for transmitting information via a radio carrier wave.

In amplitude modulation, the carrier signal is a single tone with frequency, f_c, i.e.

$$AM = A\cos(2\pi f_c t) \tag{1}$$

Its amplitude A is modulated by lower frequency modulation signal (or message signal in communication), $m(t)$.

$$A(t) = A_0[1 + m(t)] \tag{2}$$

Without losing the generality, we assume the modulation signal is a single frequency tone with frequency f_m, and then we can have

$$AM = A_0[1 + \beta\cos(2\pi f_m t)]\cos(2\pi f_c t) \tag{3}$$

where β is called the modulation index and is less than 1 in radio communications for better signal recovery. Expanding Eq.(3), we have

$$\begin{aligned} AM &= A_0\cos(2\pi f_c t) + \\ &+ \frac{A_0\beta}{2}\cos[2\pi(f_c + f_m)t] + \frac{A_0\beta}{2}\cos[2\pi(f_c - f_m)t] \end{aligned} \tag{4}$$

As a result, this amplitude modulation produces multiple frequency components, i.e., the carrier frequency f_c and side bands $f_c - f_m$ and $f_c + f_m$ which are slightly above and below carrier frequency. The amplitude of the carrier frequency depends on the value of the modulation index β, as seen in Figure 5 for modulation index less than 1 and in Figure 6 for modulation index greater than 1. Amplitude modulation that results in two sidebands and a carrier is often called double sideband amplitude modulation (DSB-AM).

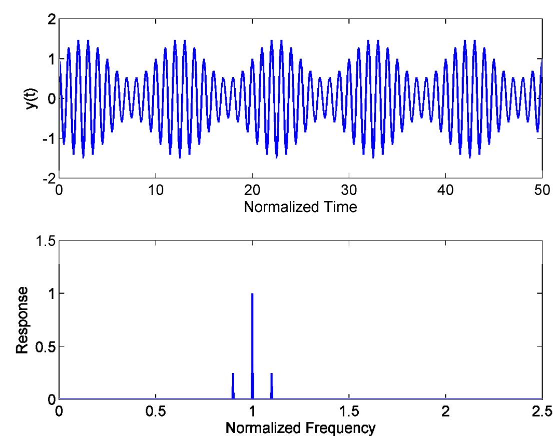


Figure 5 Amplitude modulation with $\beta < 1$

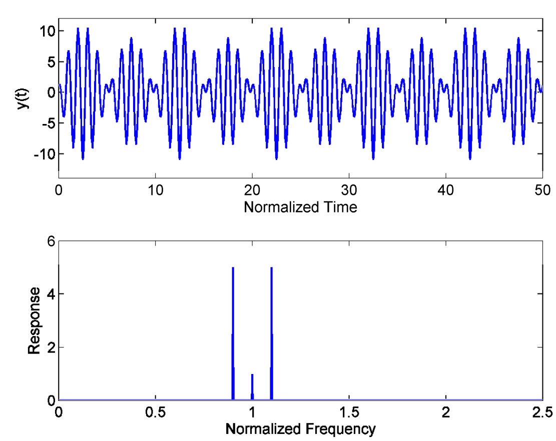


Figure 6 Amplitude modulation with $\beta > 1$

Ideally, the carrier frequency should be substantially greater than the frequency of the modulated signal. If the bandwidth of $m(t)$ is **B** Hz, carrier frequency f_c has to be

greater than $2\pi B$ to avoid overlap of the spectra centered at f_c and - f_c.

AMPLITUDE MODULATION IN STAGE 2 AND 3

During the bearing fault propagation, amplitude modulation phenomenon could happen in both stage 2 and stage 3, with different carrier and modulation signals. In most of machine configuration, we have following relationship

Shaft Speed << BFPO/I<<Structural resonance freq (N)

Therefore, in stage 2, structural resonance frequency (denoted as N in the figure below) acts as carrier frequency and bearing defect frequency (assuming we only look into outer race defect for now and it is denoted as BPFO in the figure below) acts as modulated frequency. To extract bearing defect frequency out, demodulation should be conducted around structural resonance region.

In stage 3, comparing to the shaft speed (denoted as 1X in the figure below), bearing defect frequencies are no longer a low frequency component. Instead of be modulated, it now become a carrier signal and shaft speed is modulated. Directly examining the FFT spectrum around the bearing defect characteristic frequency region will reveal BPFO+/- shaft speed components.

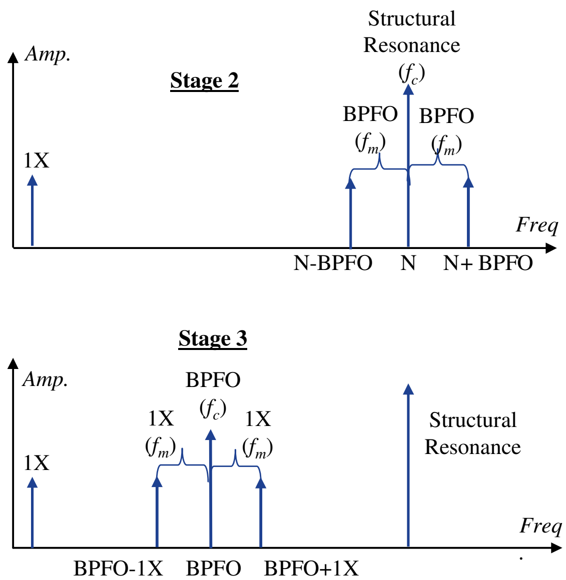


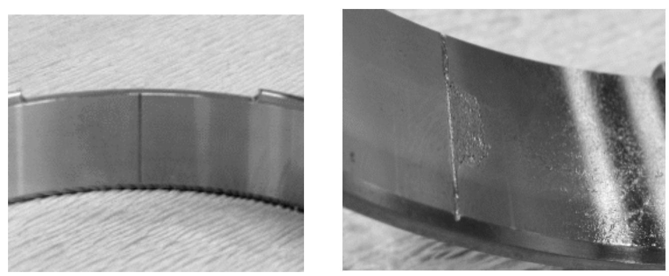
Figure 7. Different amplitude modulation phenomena at stage 2 and 3

In summary, detecting the bearing defect at its early stage needs high frequency based techniques. In stage 2, enveloping analysis is usually used to demodulate bearing defect signal from the structural resonance which is excited by the defect. In stage 3, bearing defect frequency components will show up on FFT spectrum directly, mostly like they will act as carrier frequencies for shaft speed frequency caused by unbalance or misalignment etc. As shown in equation(4), the existence of carrier frequency component depends on the value of modulation index β. It is possible that bearing defect frequencies vanished from the spectrum entirely and only side bands of defect frequency pulse and minus shaft speed present.

Thus, Detecting bearing defect at stage 3 requires much lower data sampling rate than at stage 2. But the trade-off is the defection latency, in theory, comparing to 5~10% remaining useful life in stage 2, stage 3 usually only has 1~5% left.

EXPERIMENT

An experiment was conducted on a special designed aircraft engine bearing rig to investigate the rolling element bearing prognostic techniques. A notch defect was created on the outer race of a roller bearing by an electrical discharge machine. After 22 hours total run time (TRT), the inspection discovered a spall size of 0.066 square inch. Vibration data was collected with sample rate of 200KHz throughout the experiment, and both enveloping analysis and direct FFT methods were applied to examine their effectiveness at various fault stages.



(a) (b)

Figure 8 Outer race of the testing roller bearing (a) before the test, an EDM notch was created (b) after 22 hours operation, a spall was generated along the downstream direction

The noise level of the test rig was very high due to complicated mechanical structure and dynamics. In order to differentiate bearing defect related frequency components from other source of vibration, transient data is used for analysis and time-frequency plots[12] were presented in following analysis. Time-frequency analysis illustrates the relationship between BPFO and variable shaft speed. Only the frequency component following a special multiple of the shaft speed is indeed outer race defect related (denoted as red square mark in following figures). Referring to the equation (3) and substitute with dimensions of the testing bearing, we have:

$$f_{OR} = \mathrm{BPFO} = 14.83 * RPM$$

The idle shaft speed for the rig was about 7200RPM which led to the BPFO in the range of 1780Hz. After examining multiple possible frequency regions, [15KHz 25kHz] was identified as the potential structural resonance on which the band pass filtering was conducted. Time frequency analysis for both enveloped signal (after bandpass filtering and

the Hilbert transform) and raw signal focused on the range of [0 2500Hz].

Both enveloping and FFT analysis were conducted at different TRT. Figure 9 depicted the speed profile of the data segment collected at TRT of 8.5 hours. Four seconds transient data was collected. The BFPO components showed up on enveloped signal as early as 8.5 TRT hours, as labeled as red squares in Figure 10. A side band of BFPO minus shaft speed was also visible from the time frequency plot of enveloped signal, as labeled as asterisk in Figure 10 . FFT analysis on raw vibration signal, however, did not show any defect related frequency components (Figure 11).

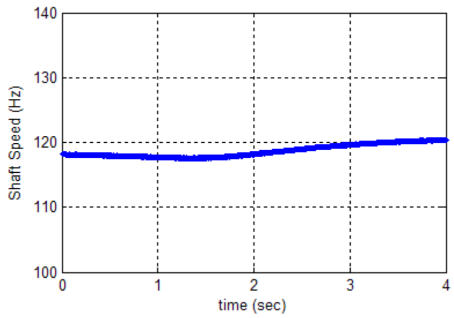


Figure 9 Shaft speed profile of the data collected at TRT 8.5 hours

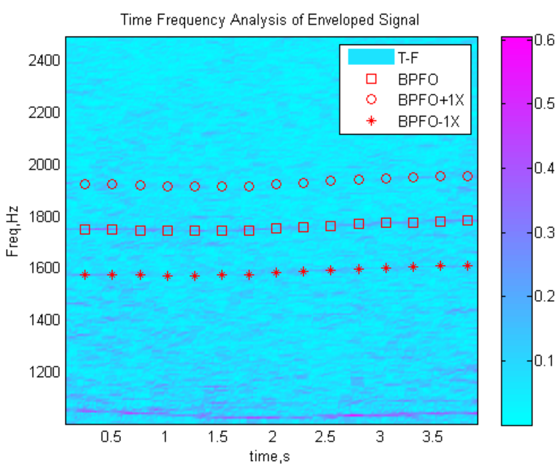


Figure 10 Time frequency plot of the enveloped signal at TRT 8.5 hours

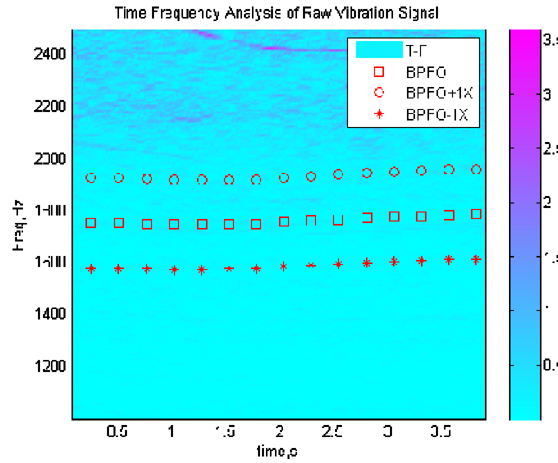


Figure 11 Time frequency plot of the raw signal at TRT of 8.5 hours

Only until after TRT was at 16.23 hours did raw vibration signal start to have BPFO component presented. Figure 12 depicted the speed profile of the data segment collected at TRT of 16.23 hours. As shown in Figure 13, two asymmetric sidebands were presented on the time frequency plane, with BPFO+1X much greater than BPFO-1X. The amplitude of the sidebands might not necessary be the same because one side maybe closer to the structural resonance than the other, and therefore was amplified.

Another interesting observation from Figure 13 was the BPFO, the carrier frequency, was not presented at all. As explained in previous session, that may due to a large modulation index involved. A defect would very likely be missed if the detection algorithm had been designed *only* looking for the BPFO component,

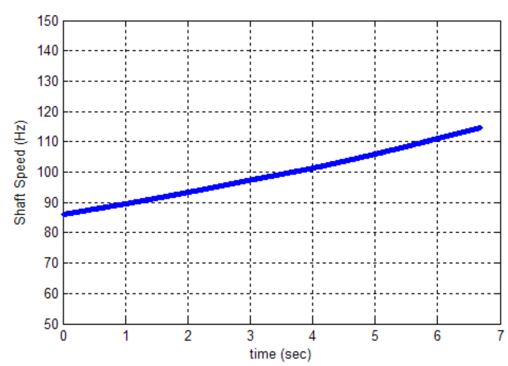


Figure 12 Shaft speed profile of the data collected at TRT of 16.23 hours

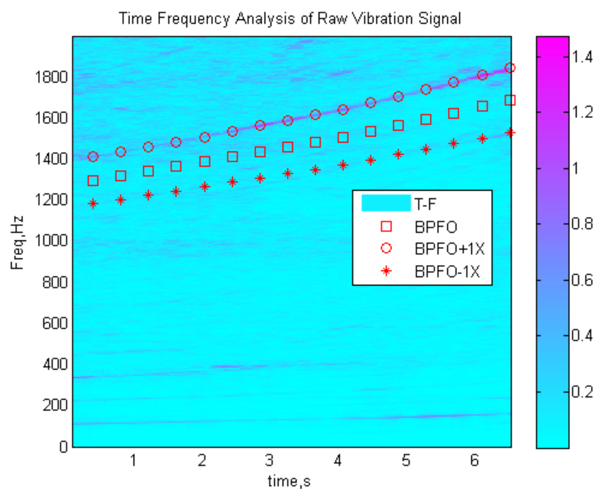


Figure 13 Time frequency plot of raw signal at TRT of 16.23 hours

To further investigate the amplitude modulation phenomenon, same analysis was repeated on another transient data collected at TRT of 21 hours, a much later stage of the test. Figure 14 depicted the speed profile of the data segment. Data was collected during the rig start-up. Figure 15 showed the time frequency analysis result for the raw vibration signal. Again, two sidebands of BPFO became very visible when the shaft speed approached operating speed. The carrier frequency itself, the BPFO, didn't present on the spectrum.

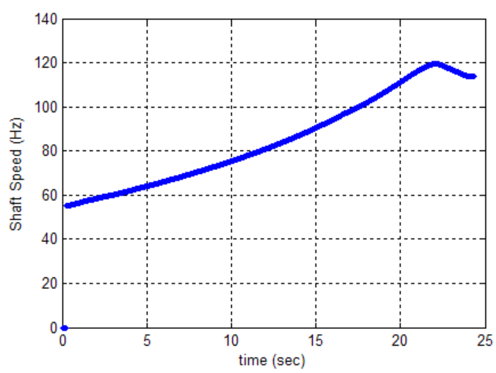


Figure 14 Shaft speed profile of the data collected at TRT of 21 hours, rig start-up process

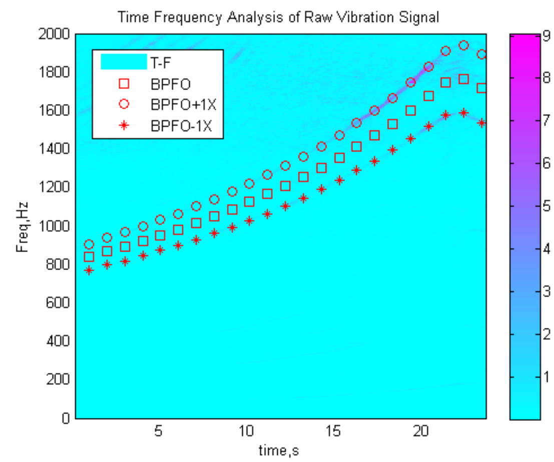


Figure 15 Time frequency plot of the raw signal at TRT of 21 hours

From this experiment, enveloping analysis was able to detect the bearing spall 7.73 hours earlier than the FFT method. If we assume the spall initiated at the beginning of the test and its propagation follows a linear rate, minimal detectable spall size for enveloping analysis is almost half of the size for the FFT method (Figure 16).

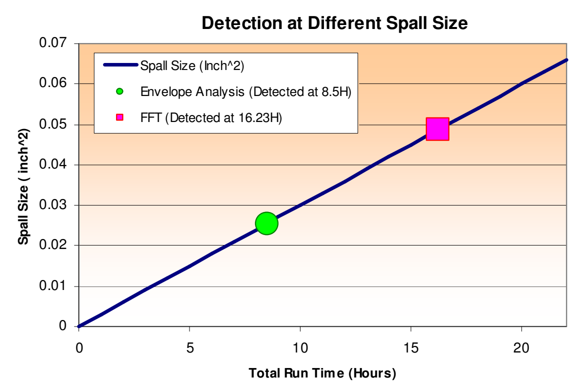


Figure 16 Comparison of detection methods vs. detectable spall sizes

However, early detection was achieved by the cost of higher data sampling rate. As mentioned above, enveloping analysis needs to be conducted on higher frequency region. In this experiment, minimal sampling rate to perform enveloping analysis is 50KHz (two times of the upper cutoff frequency of the bandpass filter-25KHz), whereas the FFT method only requires a sampling rate of 5KHz to cover the frequency range of interests. The high sampling rate requirement has several practical implications that should not be overlooked during prognostic system design, such as higher cost, more computational power, memory, and storage demanded, and hardware complexity etc.

In an onboard fault detection application, especially for critical applications such as the aircraft engines, detecting bearing spall as early as possible is of significant meanings. But an algorithm shouldn't be designed without considering hardware constraints. Not only the weight of the added on

instruments need to be cautiously controlled, but also the hardware complexity and computational demand have to be carefully considered. Computational power ought to be carefully prioritized and distributed among multiple processes to ensure the safety of the critical tasks such as flight and engine controls. A final decision should be made based on trade-off analysis of hardware requirements and tolerance of detection latency.

CONCLUSION AND DISCUSSION

This paper compared two rolling element bearing diagnostics techniques, FFT spectrum analysis and enveloping analysis, using test data from an aircraft engine test rig. Comparison was focused on the time of detection and data requirement. The results provided a reference on future on-board bearing prognostic algorithm adoption.

Experimental results verified the rolling element bearing four stage failure model, and confirmed two kinds of amplitude modulation phenomena at different stages of the failure. Bearing defect frequency was modulated by the structural resonance at early stage, whereas in the late stage, it acts as carrier frequency for shaft speed harmonics.

It also demonstrated that envelope analysis was able to detect bearing defect much earlier than spectrum analysis, but it requires a higher data sampling rate. Defect characteristic frequency eventually shows up in low frequency spectrum at the late stage of the failure and by that time the FFT analysis would be able to detect it.

Due to the amplitude modulation effect, the bearing defect characteristic frequency components itself might not show up on spectrum depending on the magnitude of unbalance, even at the late stage of the failure. Instead, it could present as a sidebands of shaft speed harmonics. A defect would very likely be missed if the detection algorithm had been designed *only* looking for the characteristic frequency components. Further researches on how unbalance influences the presence of characteristic frequency components are needed to better understand their relationship.

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